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SHORTER COMMUNICATIONS

ON THE VELOCITIES OF RISE OF ACCELERATING AND DECELERATING PARTICLES

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ACCELERATION and deceleration of bubbles rising in a liquid medium is rather common in gas–liquid mass transfer as well as in boiling and condensation heat transfer, where the bubble diameter changes during the heat- or mass-transfer process. Since direct measurement of point velocities are quite elaborate, it is common practice to measure the average velocities of rise over large pool depths. This, however, may result in errors of some ten per cent in comparatively slow mass-transfer operations. Even larger errors may be encountered in heat-transfer studies where the bubble diameter may change much faster.

In their recently published study of mass-transfer coefficients, velocities and shapes of carbon dioxide bubbles in free rise through distilled water, Calderbank and Lochiel [1] have noted that the velocities of bubbles between 0·5 and 0·8 cm in diameter fall noticeably below the values of Haberman and Morton [2] for velocities with no wall effects. Similar deviations were shown by the data of Leonard [3] and Uno and Kintner [4]. These deviations were correctly interpreted by Calderbank and Lochiel as being due to the fact that while theirs, as well as Leonard's and Uno and Kintner's, velocities were integrated values, determined as averages over large pool depths, the values of Haberman and Morton are differential point values.

The experimental technique used by Calderbank and Lochiel was mainly designed to obtain bubble cross sections, and no point velocities were measured to substantiate their suggested explanation.

It is the object of this note to present a rather extreme, though practical case where use of the average velocity may not only be erroneous quantitatively but also qualitatively in as much as it may actually obscure the actual physical phenomena taking place. This is especially true for small bubbles. The data were obtained in our study of single vapor bubbles condensing to form a liquid immiscible with the surrounding liquid medium [5].

In Fig. 1 the instantaneous velocities of an isopentane vapor bubble rising while condensing in distilled water in a 54 mm diameter column are presented. The data points represent averages of two similar runs. The initial equivalent (spherical) diameter is about 3·85 mm and the final diameter is about 1·06 mm. The temperature difference between the water and the atmospheric boiling point of isopentane for this run is 1·45 degC. (The true temperature difference at the point of bubble release is 1·93 degC). It is evident that the released bubble accelerates and reaches a maximum velocity corresponding to that of a gas bubble. Then, after some time, the velocity decreases rather abruptly, tending to a constant value. This phenomenon is most probably associated with the partial collapse of the condensing bubble and is closely related to the rather sharp decrease observed in the instanteneous heat-transfer coefficient [5]. The initial velocity and that during the collapse are presented by dotted lines since a relatively low speed ciné-camera (50–60 fps) was used in this study and exact measurements of these ranges were

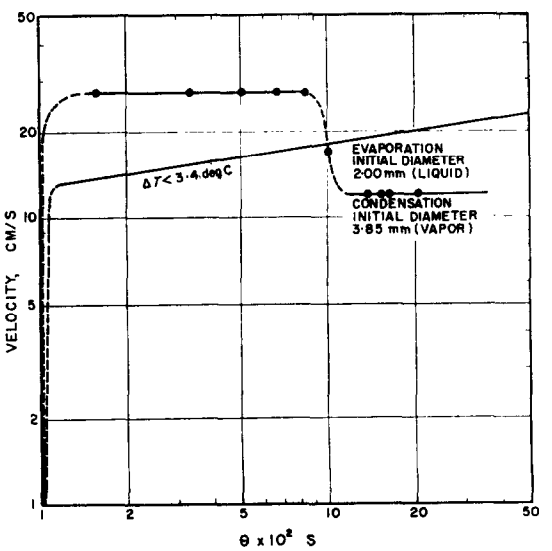

FIG. 1. Velocity of rise of condensing and evaporating bubbles in an immiscible liquid.

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not possible. Similar characteristics were observed with somewhat larger bubbles of 5·5 mm initial equivalent diameter, where the asymptotic final velocity was somewhat higher (14·5 cm/s). It is noteworthy that condensation was not complete in any of the runs discussed. The maximum amount of vapor in the final stages is estimated at about ten per cent (weight) of the original vapor in the bubble. We have also noted that whereas the collapse time was rather consistent in the small bubbles (about 0·08 s after release) that of the larger size bubbles was more erratic, varying between 0·07 and 0·12 s after release. This non-uniform behavior of the larger bubbles is most probably associated with the greater variety of bubble shapes due to larger deformations of the larger bubbles.

The results obtained indicated that smoother velocity curves with less abrupt velocity changes would be obtained with larger bubbles under similar temperature driving forces. This observation is partly substantiated by the data obtained during the complementary study of single pentane drops evaporating in water which was at some temperature above the pentane normal boiling point. There the velocity increased gradually with vapor content [6]. One of these curves obtained for drops with initial diameter of about 2 mm is included in Fig. 1. The asymptotic constant velocity corresponding to that of the fully evaporated droplet (final diameter about 12 mm) was approximately 25 cm/s (not shown here). Generally the deviation between integrated, or average velocities, and differential point velocities seemed to decrease with increasing bubble diameter. This is consistent with the results obtained by Calderbank and Lochiel in their absorption studies. It is evident, however, that where asymptotic final values are reached, averaging the velocities over an arbitrary pool height or using the asymptotic value would be quite erroneous. For instance, using the asymptotic velocity as the representative value would introduce an error of about sixty per cent with respect to the velocity averaged over the condensation height corresponding to Fig. 1.

It is also interesting to note that no apparent effect of the temperature driving force on the velocity was noted in the condensation study where rather small temperature differences (up to 3·5 degC) were used. However, the velocity, averaged over the height required for complete evaporation, was found to decrease slightly with increase of the temperature gradient (up to 15 degC) with the 2 mm liquid drops (initial diameter) evaporating in sea water [6]. No effect of temperature on the velocity was noted with evaporation of larger drops with final bubble diameter of about 22 mm.

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DIFFUSION FROM A LINE SOURCE IN A TURBULENT BOUNDARY LAYER: COMPARISON OF THEORY AND EXPERIMENT

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NOMENCLATURE*

a, b, constants in the velocity profile law;
c, concentration of the diffusing matter;
C_{max}, maximum value of concentration; concentration at the wall;
C_G, concentration in the main stream;
c_f, local drag coefficient;
G, flux of the diffusing matter per unit time and per unit length of the line source;
N_{Sc}, laminar Schmidt number;
$N_{Sc,\,t}$, turbulent Schmidt number;
U_{amb}, velocity of the ambient air stream;
u^+, non-dimensional velocity;
X', distance downstream from the origin of turbulent boundary layer;
X_0', distance of the line source from the origin of turbulent boundary layer;
x, distance downstream from the source;
x^+, non-dimensional distance along a stream line;
y, distance from and normal to the wall;

* Nomenclature as in [1] and [2].